Resource Pooling for Shared Fate: Incentivizing Effort in Interdependent Security Games through Cross-investments

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***Abstract*—We consider an InterDependent Security (IDS) game with networked agents where each agent chooses an effort/investment level for securing itself. The agents are interdependent in that the state of security of one agent depends not only on its own effort but also on that of the other agents, and each agent can benefit from other agents' security investment and effort. Since the agents are interdependent, they try to take advantage of others' security investment and effort and choose to under-invest, which leads to an inefficient Nash equilibrium (NE). While this has been analyzed extensively in the literature, in this study we take a different angle. Specifically, we consider the possibility of allowing agents to pool their resources, i.e., to both invest in themselves as well as in other agents. We show that the interaction of strategic and selfish agents under resource pooling (RP) improves the agents' effort level as well as their utility as compared to a scenario without resource pooling. We show that the social welfare (total utility) at the NE of the game with resource pooling is higher than the maximum social welfare attainable in a game without resource pooling but using an optimal incentive mechanism. Furthermore, we show that while voluntary participation in this latter scenario is not generally true, it is guaranteed under resource pooling.**

I. Introduction

The increasing rate and scale of cyber crime is placing significant pressure on organizations to improve their security posture. At the same time, the interdependent nature of cyber risks means one's state of security is not just the result of one's own security practices and investments, but of others' connected to it, e.g., through attack propagation and supply chain relationships. Decision making in such a scenario has often been modeled as an interdependent security (IDS) game. IDS games have been used to study security management in computer networks [2], cloud computing [3], and Internet of Things (IoT) networks [4]. The most critical issue that arises in IDS games is free-riding where an entity under-invests in security and takes advantage of others' efforts. As a result, a Nash equilibrium (NE) in IDS games is inefficient and individuals' investment in security is below the optimum [5].

There have been a number of studies in the literature of IDS games to address the under-investment issue. In order to improve the agents' levels of investment and reduce free-riding, various incentive mechanisms have been proposed. Grossklags *et al.* [6] show that bonus and penalty based on agents' security outcome can improve network security. Khalili *et al.* [7] show that cyber insurance can be used as an incentive mechanism in IDS games, and in the presence of a quantitative security assessment (pre-screening), it is able to improve the security investment and address free-riding issue.

Ioannidis *et al.* in [8] show that a well-informed steward (e.g., a policy maker) can address the under-investment issue in IDS games through mandate. Naghizadeh *et al.* in [9] analyze the Pivotal (VCG) and Externality mechanisms (both are in the form of a taxation/subsidy mechanism) to induce socially optimal outcome in IDS games.

All of the above proposed mechanisms have to be implemented by a central entity (e.g., a social planner, policy maker, an insurer, etc.). In this study, we shall take a different approach to inducing a socially optimal outcome in this type of IDS games. Specifically, we consider the absence of such a central entity, and instead propose resource pooling (RP) as a solution to the under-investment issue in IDS games. We model resource pooling by allowing agents to have the ability to both invest in themselves as well as in other agents, so that they can choose to not only improve its own but also others' state of security. This modeling choice leads to a different IDS game, referred to as the RP-augmented IDS game, or simply RP-IDS game. In practice, exerting efforts on other agents' behalf has context dependent interpretations, such as providing product/service discounts to customers by a service provider, as well as funding open source development. This somewhat idealistic model is then extended to a more realistic, community-based RP-IDS game where agents pool resources within a defined community/subset of agents.

Note that both IDS game and RP-IDS are non-cooperative games where agents selfishly choose their action to maximize their own utility. Thus our model is different from existing literature on cooperative games [10] where the players form coalitions and choose an action to maximize the utility of the coalition that they belong to. A cooperative game is able to improve network security as compared to a non-cooperative scenario if the cost of forming coalition is sufficiently low, but forming coalition is not always possible due to cultural, economical, or social reasons [11].

We study the IDS game with a weighted total effort and

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quadratic cost model under two scenarios: (i) no RP (the original IDS game), where each agent exerts effort only to improve his own security; and (ii) with RP (RP-IDS), where selfish agents pool their resources. It is worth mentioning that while we study the notion of resource pooling in the context of IDS games, our model and results are applicable to a broader range of problems analyzing dependent relationship (e.g., network games [12]).

Our main findings are summarized as follows.

1) Both games have a unique NE. At the NE of the RP-IDS game, every agent obtains higher utility as compared to that under the NE of the IDS game.
2) The social welfare (measured by total utility) at the NE of the RP-IDS game is higher than that under the *socially optimal* outcome of the IDS game, induced by mechanisms such as VCG and externality mechanisms [9]. In other words, as a mechanism, RP outperforms these tax-based mechanisms.
3) While the VCG and externality mechanisms cannot guarantee voluntary participation while imposing budget balance [9], we show that in the RP-IDS game no agent will unilaterally opt out of resource pooling (while continue to be part of the IDS game), thereby ensuring voluntary participation.
4) We consider a community scenario where the agents are able to pool their resources within the communities that they belong to, and these communities need not be disjoint. We show this community based resource pooling is always able to incentivize agents to improve their effort levels as compared to a scenario without resource pooling.

A. Related Literature

1) Distributed Mechanism Design: Distributed mechanism framework has been proposed to induce socially optimal outcome in a distributed manner, i.e., message transmission is performed locally, and mechanism/tax functions depend on messages from neighboring agents [13]. Even though distributed mechanisms are viable options to implement the socially optimal outcome without a central planner, they still cannot be used in IDS games because they are in the form of taxation mechanism and not able to satisfy the notion of voluntary participation [9].

2) IDS games: Outside the incentive context, IDS games have been extensively studied in the literature [14], [15], [16], [17], [18]. Ann Miura-Ko *et al.* [14] consider a linear influence network and find a condition on the dependence matrix to guarantee the existence and uniqueness of the NE. Hota and Sundaram in [15] consider IDS games under behavioral probability weighting and show that security risk can be reduced by such weighting strategies. Jiang *et al.* in [16] show that the price of anarchy in an IDS game can increase with the network size regardless of security technology improvement, while a repeated security game can decrease the price of anarchy and make the resulting NE more efficient. [17] shows that the underinvestment issue similarly exists in a two-stage game model. [18] examines the relationship between risk exposure and agents' degrees in the dependence graph. Finally, the effect of network structure on the existence and uniqueness of an NE has been studied in the more general context of network games, of which IDS games are a special case, see e.g., [19], [20].

In the remainder of the paper, we present the IDS game model without RP, and the RP-IDS game model, and their associated analysis, in Sec. II and III, respectively. A number of discussions are given in Sec. VII. Sec. VIII concludes the paper.

II. Interdependent Security Game without Resource Pooling (IDS)

Consider n agents on a directed, weighted graph denoted by $\mathcal{G} = (\mathcal{V}, \mathcal{E}, X)$, where $\mathcal{V} = \{1, 2, \cdots, n\}$ is the set of n agents, $\mathcal{E} \subseteq \{(i,j) | i, j \in \mathcal{V}\}$ the set of edges between them, and $X = [x_{ij}]_{n\times n}$ the adjacency weight matrix of this graph, where $x_{ij} > 0, i \neq j,\ (i,j) \in \mathcal{E}$ is the edge weight, $x_{ij} = 0, (i,j) \notin \mathcal{E}$, and $x_{ii} = 0, i \in \mathcal{V}$. An edge $(i,j) \in \mathcal{E}$ indicates that agent i *depends on* agent j (or agent j *influences* i) with the degree of dependence given by edge weight x_{ij}. Dependence need not be symmetrical, i.e., $x_{ij} \neq x_{ji}$ in general. Agent i exerts effort $e_i \geq 0$ towards securing himself, incurring cost $b_i \cdot e_i^2$ ($b_i > 0$ a constant). Given effort profile $\boldsymbol{e} = [e_1, e_2, \cdots, e_n]^T$, agent i has utility

$$u_i(e_i, e_{-i}) = -l_i + a_i \cdot e_i + e_i \cdot (\sum_{j=1}^{n} x_{ij} e_j) - b_i \cdot e_i^2, \quad (1)$$

where e_{-i} denotes all elements in $\boldsymbol{e}$ excluding e_i, $-l_i$ a nominal loss agent i suffers without any effort, $a_i \cdot e_i$, $a_i \geq 0$, the benefit it derives from effort e_i, and $e_i \cdot x_{ij} \cdot e_j$ the benefit it derives from other agents' efforts. This last term indicates a case of positive externality between agents i and j; see e.g., [21] for IDS games with negative externalities. Second and third terms together in Equation (1) imply that with zero effort, agent i cannot benefit from other agents' efforts, i.e., it cannot solely rely on the others. This is a form of the quadratic utility function widely used in the literature of network games [19], [22] and IDS games [23], [4]; it provides a second-order approximation to higher order concave utility functions while preserving the properties of them [23]. Note that quadratic cost function $b_i \cdot e_i^2$ implies that the effort cost grows faster than the benefit derived from effort e_i. If the effort cost does not grow faster than the benefit, it is beneficial for the agents to exert an unbounded effort. The quadratic cost function avoids the model from being pathological. However, our analysis and methodology remain valid for other cost functions as long as the cost function grows faster than the benefit from the effort.

It is worth mentioning that "effort" is an abstraction and captures the level of spending in enhancing one's cybersecurity posture in IDS games, but can model other application contexts as well. It can be considered an investment in a service or a product that benefits the service provider as well as other businesses using this service/product. In the marketing context, the effort can be an investment for attracting more costumers to a specific product, which can increase the consumers of related products. For instance, if Netflix attracts more customers, it is likely that we see an increase in the number of customers who upgrade their Internet service for a better streaming experience.

The interaction of agents induces a game, denoted as $G = \{\mathcal{V}, \{u_i(.)\}_{i\in\mathcal{V}}, A = [0, +\infty)^n\}$, where A is the action space. In the rest of the paper, we shall use the terms *exerted effort*, *actions* and *security investments* interchangeably. For convenience of notation, when comparing two games given by the same $\mathcal{V}, \mathcal{E}$ but different weight matrices X_1 and X_2, we will denote the resulting games as $G(X_1)$ and $G(X_2)$, respectively. Next we analyze the equilibrium of game G.

A. Equilibrium Analysis

Let $Br_i(e_{-i})$ denote the best response function of agent i. Using the first order condition we have

$$Br_i(e_{-i}) = \arg\max_{e\geq 0}\ u_i(e, e_{-i}) = \frac{a_i}{2b_i} + \frac{1}{2b_i}\sum_{j=1}^{n} x_{ij}e_j\ . \quad (2)$$

We will primarily focus on pure strategy Nash equilibrium (NE), and for simplicity of exposition for the rest of the paper Nash equilibrium refers to a pure strategy NE. [1] An NE is the fixed point of the best response mapping. Let $\hat{\boldsymbol{e}}$ denote the agents' effort at the NE of game G; then $\hat{\boldsymbol{e}}$ satisfies the following equations:

$$2b_i\hat{e}_i - \sum_{j=1}^{n} x_{ij}\hat{e}_j = a_i,\ \forall i \in \mathcal{V} \text{ or } (2\cdot B - X)\cdot\hat{\boldsymbol{e}} = \boldsymbol{a}, \quad (3)$$

where B is a matrix with b_i's on its main diagonal and zeros everywhere else, and $\boldsymbol{a} = [a_1, a_2, \cdots, a_n]^T$.

We make the following assumption on cost b_i to ensure that the effort levels are bounded at the NE. More discussion on this assumption is provided in Section VII-A.

Assumption 1: $2b_i > \sum_{j=1}^{n} x_{ij},\ \forall i \in \mathcal{V}$.

Assumption 1 states the technical requirement that the cost of effort (b_i) should be sufficiently large; otherwise, the utility functions and the optimal effort levels may be unbounded. A physical interpretation of this is that the agents gain more by lowering their effort (as the cost of effort is high) and relying on others. More on the importance of this assumption is discussed in Section VII-A.

Under Assumption 1, we have the following lemma on the best response mapping and the NE of game G.

Theorem 1: Under Assumption 1, matrix $(2B - X)$ is invertible and $\hat{\boldsymbol{e}} = (2\cdot B - X)^{-1}\cdot\boldsymbol{a}$ is the unique NE of game G.

Proof. See Appendix. ■

Note that Theorem 1 holds for any non-negative vector $\boldsymbol{a}$, which leads to the following corollary.

Corollary 1: Under Assumption 1, all entries of $(2\cdot B - X)^{-1}$ are non-negative. Furthermore, let X and $\tilde{X}$ be two adjacency matrices over the same $\mathcal{V}$ and $\mathcal{E}$. Consider the games $G(X)$ and $G(X+\tilde{X})$, and their respective NE $\hat{\boldsymbol{e}}$ and $\tilde{\boldsymbol{e}}$. If $2b_i \geq \sum_{j=1}^{n}[x_{ij} + \tilde{x}_{ij}]$, then $\tilde{\boldsymbol{e}} \succeq \hat{\boldsymbol{e}}$.[2] In other words, agents exert higher effort at the NE given a stronger degree of dependence among agents.

Proof. See Appendix. ■

B. Socially optimal outcome

We now consider the socially optimal effort levels for the IDS game. Denote by $\boldsymbol{e}^* = [e_1^*, e_2^*, \cdots, e_n^*]$, the socially optimal effort profile maximizes the total utility:

$$\boldsymbol{e}^* \in \arg\max_{\boldsymbol{e}\in A}\sum_{i=1}^{n} u_i(e_i, e_{-i})\ . \quad (4)$$

To ensure the existence of a socially optimal strategy, we make the following assumption.

Assumption 2: $2b_i > \sum_{j=1}^{n}[x_{ij} + x_{ji}]\ ,\ \forall i \in \mathcal{V}$.

Similar to Assumption 1, Assumption 2 also implies that the agents gain more by lowering their effort and relying on others' effort. More on this is discussed in Section VII-A.

Theorem 2: Let $\hat{\boldsymbol{e}}$ be the effort level at the NE of game G and $\boldsymbol{e}^*$ be the socially optimal effort level. Then under Assumption 2 we have:

1) $\boldsymbol{e}^* = (2B - X - X^T)^{-1}\cdot\boldsymbol{a}$;
2) $e_i^* \geq \hat{e}_i,\ \forall i$.

That is, every agent exerts higher effort at the socially optimal solution compared to the NE.

Proof. See Appendix. ■

Remark: The above shows that the socially optimal effort profile of game $G(X)$, given by $\boldsymbol{e}^* = (2B - X - X^T)^{-1}\cdot\boldsymbol{a}$, also happens to be the NE of game $G(X + X^T)$. Also note that for game $G(X)$, while the total utility under $\boldsymbol{e}^*$ is higher than that under the NE $\hat{\boldsymbol{e}}$, this may or may not be true for an agent's individual utility. In other words, we always have $\sum_{i=1}^{n} u_i(\boldsymbol{e}^*) \geq \sum_{i=1}^{n} u_i(\hat{\boldsymbol{e}})$, but it is possible that $u_i(\boldsymbol{e}^*) < u_i(\hat{\boldsymbol{e}})$ for some i.

In the next section, we will examine the impact of introducing resource pooling as a mechanism to improve agents' effort and social welfare.

III. Interdependent Security Game with Resource Pooling (RP-IDS)

Consider the same IDS game setting. Let $\boldsymbol{e}_i = [e_{i1}, e_{i2}, \cdots, e_{in}]^T$ be the action of agent i where $e_{ij} \geq 0$ denotes the effort exerted by agent i on behalf of agent j. Moreover, agent i incurs cost $b_j \cdot e_{ij}^2$ by exerting effort e_{ij} on behalf of agent j. Let $E = [\boldsymbol{e}_1, \boldsymbol{e}_2, \cdots, \boldsymbol{e}_n]^T$ be an $n\times n$ matrix that denotes the effort profile, and let $E_i = \sum_{j=1}^{n} e_{ji}$ denote the total effort exerted on behalf of agent i. Agent i's utility given profile E is:

$$\begin{aligned} v_i(\boldsymbol{e}_i, \boldsymbol{e}_{-i}) &= -l_i + a_i(\sum_{j=1}^{n} e_{ji}) - \sum_{k=1}^{n} b_k\cdot e_{ik}^2 \\ &+ (\sum_{j=1}^{n} e_{ji})\cdot\left(\sum_{k=1}^{n} x_{ik}\cdot(\sum_{r=1}^{n} e_{rk})\right) \\ &= -l_i + a_iE_i + E_i\cdot\sum_{j=1}^{n} x_{ij}E_j - \sum_{k=1}^{n} b_k\cdot e_{ik}^2. \end{aligned}$$

Our resource pooling model implies that each agent can exert effort on behalf of other agents and make an investment in others' security to improve their security posture.

[1] There is no mixed strategy Nash equilibrium for the game G. We provide a discussion on mixed strategies in the Appendix.

[2] $\boldsymbol{\nu} = [\nu_1\cdots\nu_n]^T \succeq \boldsymbol{\theta} = [\theta_1\cdots\theta_n]^T$ means that $\nu_i \geq \theta_i,\ \forall i$.

Resource pooling can have different interpretations in other contexts. For instance, in the marketing context, it can be considered an investment in a joint product/service where both providers/companies benefit if a customer purchases their product.

The interaction of agents induces the RP-IDS game $G_{rp} = \left\{\mathcal{V}, \{v_i\}_{i\in\mathcal{V}}, A_{rp} = [0,+\infty)^{n^2}\right\}$, where A_{rp} is the action space under resource pooling. By first order condition the best response function of agent i satisfies the following:

$$\begin{aligned} \boldsymbol{e}_i &= Br_i(\boldsymbol{e}_{-i}) \\ e_{ii} &= \frac{a_i}{2b_i} + \frac{\sum_{k=1}^n x_{ik}\cdot E_k}{2b_i} \\ e_{ij} &= \frac{x_{ij}\cdot E_i}{2b_j},\ \forall j\neq i \end{aligned} \tag{5}$$

Let $\hat{E} = [\hat{e}_{ij}]_{n\times n}$ be the NE of game G_{rp} and $\hat{E}_i = \sum_{j=1}^n \hat{e}_{ji}$ the total effort exerted on behalf of agent i at the NE. We have the following lemma on effort profile $\hat{E}$.

Lemma 1: Assume that game G_{rp} has at least one Nash equilibrium. The effort profile $\hat{E}$ at the NE satisfies the following system of equations,

$$(2B - X - X^T)\cdot\left[\hat{E}_1,\cdots,\hat{E}_n\right]^T = \boldsymbol{a}\ .$$

Proof. As $\hat{E}$ is the fixed point of the best response mapping, we have,

$$\begin{aligned} \hat{e}_{ii} &= \frac{a_i}{2b_i} + \frac{\sum_{k=1}^n x_{ik}\cdot \hat{E}_k}{2b_i} \\ \hat{e}_{ji} &= \frac{x_{ji}\cdot \hat{E}_j}{2b_i}\ \forall j\neq i \implies \end{aligned}$$

by adding above equations:

$$\begin{aligned} 2b_i\cdot\hat{E}_i &= a_i + \sum_{j=1}^n (x_{ij}+x_{ji})\hat{E}_j \quad \forall i\in\mathcal{V} \\ \implies \boldsymbol{a} &= (2B - X - X^T)\cdot\left[\hat{E}_1,\cdots,\hat{E}_n\right]^T \end{aligned} \tag{6}$$

■

Theorem 3: Under Assumption 2, $(2B - X - X^T)$ is invertible and game G_{rp} has a unique NE given as follows:

$$\begin{aligned} \left[\hat{E}_1,\cdots,\hat{E}_n\right]^T &= (2B - X - X^T)^{-1}\cdot\boldsymbol{a} \\ \hat{e}_{ii} &= \frac{a_i}{2b_i} + \frac{\sum_{k=1}^n x_{ik}\cdot \hat{E}_k}{2b_i} \\ \hat{e}_{ij} &= \frac{x_{ij}\cdot \hat{E}_i}{2b_j},\ \forall j\neq i \end{aligned} \tag{7}$$

Proof. Similar to the proof of Theorem 1, we can show that if $2b_i \geq \sum_{j=1}^n x_{ij} + x_{ji}, \forall i$, then all eigenvalues of matrix $(2B-X-X^T)$ are non-zero. Therefore, matrix $(2B-X-X^T)$ is invertible. By Corollary 1, all entries of $(2B-X-X^T)^{-1}$ are non-negative and $\left[\hat{E}_1\cdots\hat{E}_n\right]^T = (2B-X-X^T)^{-1}\cdot\boldsymbol{a}$ is a non-negative vector. Moreover, by best response mapping provided in Equation (5), we know that $\hat{e}_{ij}$ can be calculated by the following,

$$\begin{aligned} \hat{e}_{ii} &= \frac{a_i}{2b_i} + \frac{\sum_{k=1}^n x_{ik}\cdot \hat{E}_k}{2b_i} \geq 0 \\ \hat{e}_{ij} &= \frac{x_{ij}\cdot \hat{E}_i}{2b_j} \geq 0,\ \forall j\neq i \end{aligned} \tag{8}$$

Therefore, the fixed point of the best response mapping is non-negative and unique, implying the NE of game G_{rp} is unique and can be found by Equation (7). ■

Remark: It is worth pointing out that for the same weight matrix X, the *total* effort exerted by each agent, $[\hat{E}_1,\ \hat{E}_2,\cdots,\hat{E}_n]$, at the NE of the RP-IDS game G_{rp} is the same as the socially optimal effort of the IDS game G. That is,

$$\begin{aligned} \left[\hat{E}_1,\cdots,\hat{E}_n\right]^T &= (2B - X - X^T)^{-1}\cdot\boldsymbol{a} \\ &= \boldsymbol{e}^* \underbrace{\succeq}_{\text{By Theorem 2}} \hat{\boldsymbol{e}}\ . \end{aligned} \tag{9}$$

In other words, the introduction of resource pooling incentivizes agents to boost their effort to the socially optimal levels for game G. Note that the game G_{rp} has its own socially optimal solution as we discuss in Section VII-B.

Next we show that every agent at the NE of game G_{rp} obtains a higher utility than that attained at the NE of game G, i.e., resource pooling improves the utility for all agents.

Theorem 4: Let $\hat{E} = [\hat{e}_{ij}]_{n\times n}$ be the NE of G_{rp} and $\hat{\boldsymbol{e}}$ be the effort profile at the NE of game G. Under Assumption 2, we have:

$$v_i(\hat{E}) \geq u_i(\hat{\boldsymbol{e}}),\quad \forall i\in\mathcal{V}\ . \tag{10}$$

Proof. Let $\tilde{\boldsymbol{e}}_i$ be a vector with length n and all its elements are zero except entry i which is equal to $\hat{e}_i$ (effort level of agent i at NE of game G). By definition of Nash equilibrium we have,

$$v_i(\hat{E}) \geq v_i(\tilde{\boldsymbol{e}}_i, \hat{\boldsymbol{e}}_{-i}). \tag{11}$$

As $\hat{E}_i \geq \hat{e}_i,\ \forall i$, by Equations (7) and (3) we have $\hat{e}_{ii}\geq\hat{e}_i$. Moreover,

$$\begin{aligned} v_i(\tilde{\boldsymbol{e}}_i,\hat{\boldsymbol{e}}_{-i}) = -l_i + a_i\cdot\hat{e}_i + a_i\sum_{k\neq i}\hat{e}_{ki} - b_i\cdot(\hat{e}_i)^2 \\ +(\hat{e}_i + \sum_{k\neq i}\hat{e}_{ki})\cdot\sum_{j=1}^n\left(x_{ij}\cdot(\sum_{k\neq i}\hat{e}_{kj})\right) \geq \\ -l_i + a_i\cdot\hat{e}_i - b_i\cdot(\hat{e}_i)^2 + \hat{e}_i\cdot\sum_{j=1}^n x_{ij}\cdot\hat{e}_j = u_i(\hat{e}_i,\hat{e}_{-i}) \end{aligned} \tag{12}$$

By Equations (11) and (12), $v_i(\hat{E}) \geq u_i(\hat{\boldsymbol{e}})\ \forall i\in V$. ■

The following theorem shows that social welfare at the NE of game G_{rp} is higher than the maximum social welfare of game G, even though the total effort exerted by each agent is the same under both as noted earlier.

Theorem 5: Let $\hat{E}$ be the effort profile at the NE of game G_{rp} and $\boldsymbol{e}^*$ be the socially optimal effort profile in game G. Under Assumption 2 we have, $\sum_{i=1}^n v_i(\hat{E}) \geq \sum_{i=1}^n u_i(\boldsymbol{e}^*)$.

Proof.

$$\sum_{i=1}^n v_i(\hat{E}) = \sum_{i=1}^n\left(-l_i + a_i\hat{E}_i - b_i\cdot\left[\sum_{j=1}^n\hat{e}_{ji}^2\right] + \hat{E}_i\cdot\left[\sum_{j=1}^n x_{ij}\cdot\hat{E}_j\right]\right)$$

By Equation (9), $(e_i^*)^2 = \hat{E}_i^2 = (\sum_{j=1}^n\hat{e}_{ji})^2 \geq \sum_{j=1}^n(\hat{e}_{ji})^2$, and $\hat{E}_i = e_i^*$. Therefore,

$$\begin{aligned} \sum_{i=1}^n v_i(\hat{E}) &\geq \sum_{i=1}^n\left(-l_i + a_i\hat{E}_i - b_i\cdot\hat{E}_i^2 + \hat{E}_i\cdot\left[\sum_{j=1}^n x_{ij}\cdot\hat{E}_j\right]\right) \\ &= \sum_{i=1}^n u_i(\boldsymbol{e}^*). \end{aligned}$$

■

We conclude this section by highlighting the role of resource pooling in the IDS game.

- At the NE, with resource pooling (game G_{rp}) agents exert higher effort (for themselves and on others) and experience higher utility than without (game G); e.g., $\hat{E}_i \geq \hat{e}_i$, and $v_i(\hat{E}) \geq u_i(\hat{\boldsymbol{e}})$.
- Resource pooling induces agents to exert socially optimal levels of effort (under game G), while improving the social welfare as it allows more judicious spreading of efforts; e.g., $\hat{E} = \boldsymbol{e}^*$ and $\sum_{i=1}^{n} v_i(\hat{E}) \geq \sum_{i=1}^{n} u_i(\boldsymbol{e}^*)$.

IV. Best Response Dynamics for the IDS game (Game G) and the RP-IDS game (Game G_{rp})

Based on Theorem 1 and 3, we have to calculate an inverse of a matrix to find the Nash equilibrium of game G and G_{rp}. In this section, we develop an iterative best response dynamics which converges to the Nash equilibrium without calculating the matrix inversion.

A. Best Response Dynamics for IDS game (Game G)

Best response dynamics are well-known algorithms to find a Nash equilibrium iteratively. In each iteration of the best response dynamics, the action of each agent is updated based on the best response function [24]. Algorithm 1 shows the best response dynamics of game G. In the theorem below, we use results from [25] to show that Algorithm 1 converges to the NE of game G.

Theorem 6: Under assumption 1, Algorithm 1 converges to $\hat{\boldsymbol{e}}$, the Nash equilibrium of game G.

Proof. It is easy to check that under Assumption 1, the best response mapping of game G, $Br(\boldsymbol{e}) = [Br_1(\boldsymbol{e}_{-1}), \cdots, Br_n(\boldsymbol{e}_{-n})]^T = \frac{B^{-1}}{2} \cdot (\boldsymbol{a} + X \cdot \boldsymbol{e})$, is a contraction mapping. By [25], the best response dynamics shown in Algorithm 1 converges to the NE. ■

Algorithm 1 Finding Nash equilibrium for game G using best response dynamics

Input: Game parameters $(\boldsymbol{a}, X, \boldsymbol{b})$, Number of iterations $(\mathcal{T})$.
Initialization: set $e_k^{(0)} = \max_i \frac{a_i}{2b_i - \sum_{j=1}^{n} x_{ij}}$, $\forall\ k \in \mathcal{V}$
for *t= 1,2,$\cdots$, $\mathcal{T}$* **do**
  $e_i^{(t)} = \frac{a_i}{2b_i} + \frac{1}{2b_i} \sum_{j=1}^{n} x_{ij} e_j^{(t-1)}, \forall\ i \in \mathcal{V}$
end
Output: $[e_1^{(\mathcal{T})}, e_2^{(\mathcal{T})}, \cdots, e_n^{(\mathcal{T})}]$

It is easy to see that Algorithm 1 has $n \cdot (n+1)$ multiplications and n^2 additions in each iteration. Since the **for** loop runs $\mathcal{T}$ times, the complexity of Algorithm 1 is $\mathcal{O}(\mathcal{T} \times n^2)$.

B. Modified Best Response Dynamics for RP-IDS game (G_{rp})

Best response dynamics for Game G_{rp} can be computationally expensive because it updates each agent's action using the best response function given by a system of linear equations defined in Equation (5). The lower bound on the complexity of solving a linear system is $\mathcal{O}(n^2)$ [26]. Since we have to solve a linear system for each agent, we need at least $\mathcal{O}(n^3)$ computations in each iteration of the best response dynamics.

In order to avoid solving a system of linear equations for each agent in every iteration, we propose a *modified* best response dynamics which updates the total effort exerted on behalf of each agent iteratively instead of updating the action profile. Algorithm 2 shows the *modified* best response dynamics for game G_{rp}.

Theorem 7: Under Assumption 2, the output of Algorithm 2 is the effort profile of the agents at the NE of game G_{rp}. Moreover, the computational complexity of Algorithm 2 is $\mathcal{O}(\mathcal{T} \times n^2)$, where $\mathcal{T}$ is the number of iterations.

Proof. Based on Theorem 6, we know that $E_i^{(t)}, t = 0, 1, \cdots$, defined in Algorithm 2, converges to the effort of agent i at the Nash equilibrium of game $G(X + X^T)$. Moreover, we know that the total effort exerted on behalf of agent i at the NE of game G_{rp} is equal to the effort of agent i at the NE of game $G(X + X^T)$. Therefore, $E_i^{(t)}, t = 0, 1, \cdots$ converges to the total effort exerted on behalf of agent i at the NE of game G_{rp}. By Theorem 3, the output of Algorithm 2 is the equilibrium of game G_{rp}.

It is easy to see that Algorithm 2 has n^2 multiplications, $2n^2$ additions, and n divisions in each iteration. After the **for** loop, there are n^2 additions, $2n^2 - n$ multiplications, and n^2 divisions. Since we have $\mathcal{T}$ iterations, the total computational complexity is $\mathcal{O}(\mathcal{T} \times n^2)$. ■

Algorithm 2 Finding Nash equilibrium for game G_{rp} using the modified best response dynamics

Input: Game parameters $(\boldsymbol{a}, X, \boldsymbol{b})$, Number of iterations $(\mathcal{T})$.
Initialization: set $E_k^{(0)} = \max_i \frac{a_i}{2b_i - \sum_{j=1}^{n} x_{ij} + x_{ji}}$, $\forall\ k \in \mathcal{V}$
for *t= 1,2,$\cdots$, $\mathcal{T}$* **do**
  $E_i^{(t)} = \frac{a_i + \sum_{j=1}^{n} (x_{ij} + x_{ji}) E_j^{(t-1)}}{2b_i}, \forall\ i \in \mathcal{V}$
end
$\hat{e}_{kk} = \frac{a_k + \sum_{j=1}^{n} x_{kj} \cdot E_j^{(\mathcal{T})}}{2b_k}, \forall k \in \mathcal{V}$
$\hat{e}_{kk'} = \frac{x_{kk'} \cdot E_k^{\mathcal{T}}}{2b'_k}$, $\forall k \neq k'$
Output: $\hat{E} = [\hat{e}_{ij}]_{n \times n}$

V. Voluntary Participation in RP

As investment in security is a non-excludable public good, an agent can benefit even if it chooses not to participate in an incentive mechanism. As a result, designing a mechanism which incentivizes the agents to voluntarily participate and exert socially optimal effort levels is not straightforward. In [9], it was shown that taxation mechanisms (i.e., penalizing/rewarding agents based on agents' effort level) are not able to implement the socially optimal solution while guaranteeing both weak budget balance and voluntary participation. For this reason, it is important to check whether agents will voluntarily participate in resource pooling. In what follows, we first define this notion and then show that under resource pooling the voluntary participation property is satisfied.

*Definition 1 (**Voluntary Participation (VP)**):* Consider game G_{rp}^k where agent k opts out of RP and only invests in himself and nobody else invest in agent k ($e_{kj} = e_{jk} = 0,\ \forall j \neq k$), while other agents participate in RP. Let $\mathring{E} = [\mathring{e}_{ij}]_{n\times n}$ be the NE of game G^k_{rp} and $v_i(\mathring{E})$ be the utility of agent i at the NE. We say that resource pooling has the voluntary participation property with respect to agent k, if

$$v_k(\mathring{E}) \leq v_k(\hat{E}), \tag{13}$$

where $\hat{E}$ is the effort profile at the NE of game G_{rp}.[3] If the above is true for all $k \in \mathcal{V}$, then we say that resource pooling has the voluntary participation property.

The following theorem suggests that resource pooling always satisfies the VP property.

Theorem 8: If Assumption 2 holds, then agent i achieves higher utility at the NE of game G_{rp}, than his utility at the NE of game G^i_{rp} for all $i \in \mathcal{V}$. That is, resource pooling always satisfies the VP property.

Proof. See Appendix. ■

As no one has incentive to deviate from resource pooling unilaterally, resource pooling is a better way to improve social welfare as compared to taxation mechanisms which are not able to satisfy the voluntary participation and budget balance constraint simultaneously [9].

It is worth noting that resource pooling is able to satisfy a stronger notion of voluntary participation defined as follows.

*Definition 2 (**Stronger Notion of Voluntary Participation (SVP)**):* Consider game $\overline{G}^k_{rp}$ where agent k opts out of RP and only invests in himself ($e_{kj} = 0, \ \forall j \neq k$), while the other agents participate in RP and may choose to invest in agent k. In other words, while agent k chooses not to exert any effort on behalf of other agents, he may receive resources from other agents in game $\overline{G}^k_{rp}$ if it is in the other agents' self interest to do so. Let $\acute{E} = [\acute{e}_{ij}]_{n\times n}$ be the NE of game $\overline{G}^k_{rp}$ and $v_i(\acute{E})$ be the utility of agent i at the NE. We say that resource pooling has the strong voluntary participation property with respect to agent k, if

$$v_k(\acute{E}) \leq v_k(\hat{E}), \tag{14}$$

where $\hat{E}$ is the effort profile at the NE of game G_{rp}. If the above is true for all $k \in \mathcal{V}$, then we say that resource pooling has the strong voluntary participation property.

It is worth noting a crucial difference between the definition of an NE and the VP property. An NE in game G_{rp} implies that $v_k(\hat{\boldsymbol{e}}_k, \hat{\boldsymbol{e}}_{-k}) \geq v_k(\boldsymbol{e}_k, \hat{\boldsymbol{e}}_{-k}), \ \forall \boldsymbol{e}_k \in R^n$. In words, this definition says that at the NE, agent k cannot improve his utility by changing his action while other agents do not change their effort and keep the same action. On the other hand, Equations (13) and (14) imply that agent k is not able to improve his utility if he chooses not to pool his resources and the other agents best respond to his decision and choose their actions accordingly. The following theorem shows that resource pooling is able to satisfy the SVP property defined in Definition 2.

Theorem 9: If Assumption 2 holds, resource pooling always satisfies the SVP property defined in Definition 2.

Proof. See Appendix. ■

[3] Under Assumption 2, both G_{rp} and G^k_{rp} have an NE.

VI. Community Based Resource Pooling

So far we have assumed that each agent can pool his resources with all other agents in the network. This is a useful benchmark but not a very realistic scenario. We next consider a more realistic setting where each agent is able to pool resources within a defined community. The notion of community-based resource pooling captures situations such as *co-branding partnerships*, where two or more separate brands work together to provide services and products for their collective customers. Prime examples of such partnerships include that among Starbucks, Spotify, and Uber [27], between Amazon and American Express (Amex) [28], and between Apple and MasterCard [29]. In all these examples, one can think of the effort as the amount of marketing investment to capture more customers. Co-branding allows one company to capture customers for another. The interdependent nature also arises from their mutual reliance on customer reach and market share: co-branding would mean the more riders Uber has, the more orders Starbucks will receive through Uber Eats app, and in the case of Amazon/Amex, the more customers Amazon has, the more will be likely to obtain/use an Amex card due to the cash-back benefits Amex offers its customers on Amazon purchases. Thus it is in Amex's interest to promote Amazon to its own customers. The opposite is also true. The more customers Amex has, the more will purchase from Amazon because of the cash-back, etc.

More formally, we assume that agents form m communities (not necessarily disjoint) and they are allowed to pool their resources within the communities they belong to. Let $C_1, \cdots, C_m$ denote the m communities, where $\cup_{k=1}^m C_k = \mathcal{V}$. Let $I(i)$ be the set of indices of the communities that agent i belongs to, i.e., $i \in \bigcap_{j\in I(i)} C_j$. Let G^c_{rp} denote the game induced by the interaction of the agents who are allowed to pool their resource within their communities. Let $\boldsymbol{e}_i = [e_{ij}], j \in \bigcup_{k\in I(i)} C_k$, be the action of agent i. Let $E_i = \sum_{j\in \bigcup_{k\in I(i)} C_k} e_{ji}$ be the total effort exerted on behalf of agent i in game G^c_{rp}. The utility of agent i is given by:

$$v_i(\boldsymbol{e}_i, \boldsymbol{e}_{-i}) = -l_i + a_i E_i + E_i \sum_{j=1}^n x_{ij} E_j - \sum_{j\in \bigcup_{k\in I(i)} C_k} b_j e^2_{ij} \ .$$

Let $\check{\boldsymbol{e}}_i = [\check{e}_{ij}], \ j \in \left(\bigcup_{k\in I(i)} C_k\right)$ be the effort profile of agent i at the NE of game G^c_{rp} and $\check{E}_i = \sum_{j\in C_{I(i)}} \check{e}_{ji}$. We have the following result.

Theorem 10: Under Assumption 2, game G^c_{rp} has a unique Nash equilibrium and the effort profile of agents at the equilibrium is given by:

$$\begin{aligned} \left[\check{E}_1, \cdots, \check{E}_n\right]^T &= (2B - X - X_c^T)^{-1} \boldsymbol{a}, \\ \check{e}_{ii} &= \frac{a_i}{2b_i} + \frac{\sum_{j=1}^n x_{ij} \check{E}_j}{2b_i}, \\ \check{e}_{ij} &= \frac{x_{ij} \cdot \check{E}_i}{2b_j}, \forall j \neq i, j \in \bigcup_{k\in I(i)} C_k \end{aligned} \tag{15}$$

where the entry (i,j) of matrix X_c is equal to x_{ij} if $j \in \bigcup_{k\in I(i)} C_k$, and zero otherwise.

Proof. See Appendix. ∎

Thus both games G and G^c_{rp} have a unique Nash equilibrium under Assumption 2. The next theorem compares the NE of games G and G^c_{rp}.

Theorem 11: Under Assumption 2, we have the following results.

- Community based resource pooling improves the effort and utility of each agent as compared to those at the NE of game G. That is,

$$\begin{aligned} \check{E}_i &\geq \hat{e}_i, \forall i, \\ v_i(\check{\boldsymbol{e}}_i, \check{\boldsymbol{e}}_{-i}) &\geq u_i(\hat{e}_i, \hat{e}_{-i}), \forall i. \end{aligned}$$

- $\check{E}_i \leq e^*_i, \forall i$. That is, the total effort exerted on behalf of each agent is less than the socially optimal effort level in game G.

Proof. See Appendix. ∎

The next theorem identifies the impact of merging two communities on agents' utilities and social welfare.

Theorem 12: Consider game $\overline{G}^c_{rp}$ a community based resource pooling game with the following communities: $C_1, C_2, \cdots, C_{m-2}, C_{m-1} \cup C_m$.

Let $\check{\bar{\boldsymbol{e}}}_i$ be the strategy of agent i at the NE of game $\overline{G}^c_{rp}$. Moreover, let $\overline{I}(i)$ be the indices of the communities that agent i belongs to in game $\overline{G}^c_{rp}$. We have, $\check{\overline{E}}_i \geq \check{E}_i, \forall i$, and $v_i(\check{\bar{\boldsymbol{e}}}_i, \check{\bar{\boldsymbol{e}}}_{-i}) \geq v_i(\check{\boldsymbol{e}}_i, \check{\boldsymbol{e}}_{-i}), \forall i$, where, $\check{\overline{E}}_i = \sum_{j \in \bigcup_{k \in \overline{I}(i)} C_k} \check{\bar{e}}_{ji}$. In other words, merging two communities improves agents' utilities as well as agents' efforts.

Proof. See Appendix. ∎

While the social welfare at the NE of game G^c_{rp} is higher than that at the NE of game G, it may or may not be higher than the maximum social welfare of game G. Next we provide a numerical example to highlight the impact of community based resource pooling. Consider a network with $n = 10$ agents and the following parameters:

$$\begin{aligned} a_i &= 1, \forall i, \quad b_i = 2, \forall i \\ x_{ij} &= \begin{cases} 1 & \text{if } j = i+1 \text{ and } i \text{ is odd.} \\ 1 & \text{if } j = i-1 \text{ and } i \text{ is even.} \\ 0 & \text{if } i = j \\ 0.1 & \text{o.w.} \end{cases} \end{aligned}$$

Without loss of generality, we will set $l_i = 0, \forall i$; as l_i is a constant, this will not affect agents' decision. Given this set of parameters, we divide the agents to $m, m = 1, \cdots, 10$ communities using spectral clustering method [30] as follows.

$$\begin{aligned} m = 1,\ C_1 &= \mathcal{V} \\ m = 2,\ C_1 &= \{1, 2, 3, 4, 9, 10\},\ C_2 = \{5, 6, 7, 8\} \\ m = 3,\ C_1 &= \{1, 2, 3, 4, 7, 8\},\ C_2 = \{5, 6, 7, 8\}, \\ C_3 &= \{9, 10\} \\ m = 4,\ C_1 &= \{1, 2\}, C_2 = \{3, 4\}, C_3 = \{5, 6, 9, 10\}, \\ C_4 &= \{7, 8\} \\ m = 5,\ C_1 &= \{1, 2\}, C_2 = \{3, 4\}, C_3 = \{5, 6\}, \\ C_4 &= \{7, 8\}, C_5 = \{9, 10\} \end{aligned}$$

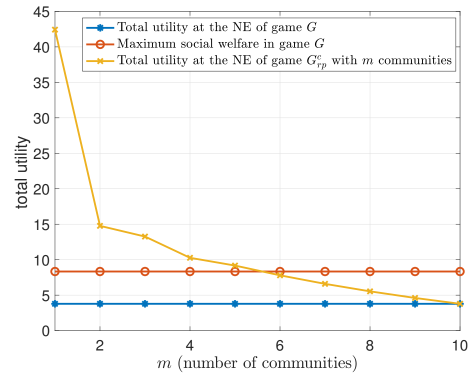

Figure 1: The total utility at the NE of game G^c_{rp} with m communities. Resource pooling within fewer and large size communities is more effective in improving social welfare.

$$\begin{aligned} m = 6,\ C_1 &= \{1\}, C_2 = \{2\}, C_3 = \{3, 4\}, C_4 = \{5, 6\}, \\ C_5 &= \{7, 8\}, C_6 = \{9, 10\} \\ m = 7,\ C_1 &= \{1, 2\}, C_2 = \{3\}, C_3 = \{4\}, \\ C_4 &= \{5, 6\}, C_5 = \{7, 8\}, C_6 = \{9\}, C_7 = \{10\} \\ m = 8,\ C_1 &= \{1\}, C_2 = \{2\}, C_3 = \{3\}, C_4 = \{4\} \\ C_5 &= \{5, 6\}, C_6 = \{7\}, C_7 = \{8\}, C_8 = \{9, 10\} \\ m = 9,\ C_1 &= \{1\}, C_2 = \{2\}, C_3 = \{3\}, C_4 = \{4\}, \\ C_5 &= \{5, 6\}, C_6 = \{7\}, C_7 = \{8\}, C_8 = \{9\}, \\ C_9 &= \{10\} \\ m = 10,\ C_k &= \{k\},\ \forall k \end{aligned}$$

It is easy to see that $m = 1$ corresponds to the case without community as studied earlier in the paper, whereas $m = 10$ corresponds to the case where resource pooling is not allowed.

Figure 1 illustrates the total utility at the NE using community based resource pooling as the number of communities m increases. These results verifies our theoretical finding that resource pooling even limited within communities always leads to higher total utility. Furthermore, we see that when $m \geq 6$, the total utility at the NE of game G^c_{rp} falls below the maximum social welfare in game G, suggesting that resource pooling is more effective with fewer and larger communities ($m \leq 5$). Figure 2 illustrates the total effort at the NE of G^c_{rp} as a function of the number of communities (m). First, we note that the total effort at the socially optimal outcome of game G is the same as the total effort at game G_{rp} ($m = 1$), as expected. Also, consistent with the previous figure, we see that the total investment decreases as a function of the number of communities m.

VII. Discussion

A. On the assumption $2b_i > \sum_{j=1}^n x_{ij}$

Throughout the analysis we have used variants of the above assumptions on the existence and uniqueness of the following:

- NE in game G: $2b_i > \sum_{j=1}^n x_{ij}, \forall i$;
- Socially optimal strategy profile in game G: $2b_i > \sum_{j=1}^n x_{ij} + x_{ji}, \forall i$;
- NE profile in game G_{rp}: $2b_i > \sum_{j=1}^n x_{ij} + x_{ji}, \forall i$.

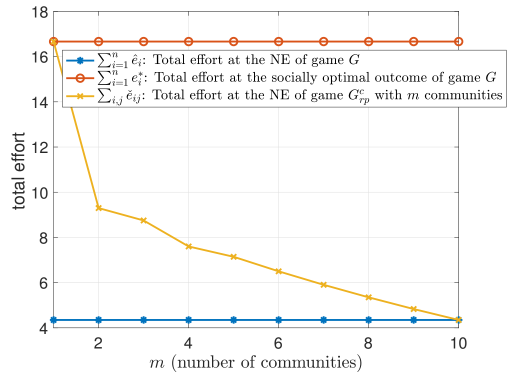

Figure 2: The total effort at the NE of game G_{rp}^c with m communities. Resource pooling within fewer and large size communities is more effective in incetivizing agents to invest more in security.

The reason behind these assumptions is to prevent the model from becoming pathological: if the cost of effort is sufficiently low, then there may not exist NE or socially optimal strategy, and it may be beneficial for the agents to exert very high effort with unbounded utility.

Example 1: Consider a network with $x_{ii} = 0$, $x_{ij} = \frac{1}{n-1}$ $\forall i, j \in V$, $i \neq j$ and $b_i = 1$. Under these parameters Assumption 2 does not hold. Moreover, set $e_i = r$, $\forall i \in V$. We have: $\sum_{i=1}^{n} u_i(\boldsymbol{e}) = \sum_{i=1}^{n}(-l_i + (r)a_i - b_i \cdot r^2 + r^2 \sum_{j=1}^{n} x_{ij}) = -\sum_{i=1}^{n} l_i + r \cdot \sum_{i=1}^{n} a_i$, which is a linear function in r and is unbounded. In this case the socially optimal effort does not exist.

B. On the socially optimal outcome of game G_{rp}

While the NE of the RP-IDS game G_{rp} achieves socially optimal levels of effort defined for the IDS game G, the introduction of resource pooling means that each agent now has a bigger action space, thereby giving rise to a different social optimum for this new game. We next show how this new optimum can be computed.

Let $E^* = [e_{ij}^*]_{n\times n}$ be the socially optimal effort profile for the RP-IDS game:

$$E^* = \arg \max_{E\in R_+^{n\times n}} \sum_{i=1}^{n} v_i(E)$$
$$= \arg \max_{E\in R_+^{n\times n}} \sum_{i=1}^{n} \left[-l_i + a_i E_i - b_i (\sum_{j=1}^{n} e_{ji}^2) + E_i \sum_{j=1}^{n} x_{ij} E_j \right].$$

The assumption below ensures the existence of a solution.

Assumption 3: $2b_i > n \cdot \sum_{j=1}^{n}(x_{ij} + x_{ji}), \ \forall i \in \mathcal{V}$

Under Assumption 3, it is easy to check that $g(E) = \sum_{i=1}^{n} v_i(E)$ is strictly concave in E. By the first order condition, E^* satisfies the following:

$$\frac{\partial g(E)}{\partial e_{ki}}|_{E=E^*} = a_i - 2b_i e_{ki}^* + \sum_{j=1}^{n}(x_{ij} + x_{ji}) \cdot E_j^* = 0 \ \forall k,$$
$$\implies n \cdot a_i - 2b_i E_i^* + n \cdot \sum_{j=1}^{n}(x_{ij} + x_{ji}) \cdot E_j^* = 0, \ \forall i \in \mathcal{V},$$
$$\implies (2B - n \cdot (X + X^T)) \cdot \left[E_1^*, \cdots, E_n^*\right]^T = n \cdot \boldsymbol{a} \ . \quad (16)$$

Again we can show $(2B - n \cdot (X + X^T))$ is invertible under Assumption 3. Thus the optimal outcome E^* is given by:

$$\left[E_1^*, \cdots, E_n^*\right]^T = n \cdot (2B - n \cdot (X + X^T))^{-1} \cdot \boldsymbol{a}$$
$$e_{ki}^* = \frac{a_i}{2b_i} + \frac{\sum_{j=1}^{n}(x_{ij} + x_{ji}) \cdot E_j^*}{2b_i}, \ \forall k, i \in \mathcal{V}$$

By Corollary 1, we have $E_i^* \succeq \hat{E}_i, \ \forall i$, i.e., the total effort exerted on behalf of agent i improves under the social optimum compared to that under the NE of game G_{rp}. As before, not all agents may attain higher individual utility under E^* as compared to their utility under NE effort profile $\hat{E}$.

VIII. Conclusion

We considered an IDS game with positive externality, and introduced a resource pooling augmented IDS game, the RP-IDS game to examine the effect of using resource pooling as a mechanism to incentivize higher effort levels by interdependent agents. We showed that (1) resource pooling increases the total effort exerted on behalf of each agent as compere to no resource pooling, (2) each agent experiences higher utility under resource pooling as compared to no resource pooling, (3) social welfare at the NE of the RP-IDS game is higher than the optimal social welfare under the IDS game, and (4) agents voluntarily participate in resource pooling.

One limitation of our model is the fixed community structure, where agents are not able to change the communities that they belong to. A very relevant and interesting extension of the current work is to study community based resource pooling when agents voluntarily participate in communities.

IX. Appendix

A. Proofs

Proof. [Theorem 1] Let $\boldsymbol{\nu}$ be the eigenvector of matrix $2B - X$ and λ its corresponding eigenvalue. Without loss of generality, we can assume that ν_i is the maximum element of $\boldsymbol{\nu}$ in absolute value ($|\nu_i| \geq |\nu_j|, \forall j$). Note that $|\nu_i| > 0$ by definition. The following shows that eigenvalues of $2B - X$ are non-zero and $2B - X$ is invertible:

$$(2B - X) \cdot \boldsymbol{\nu} = \lambda \cdot \boldsymbol{\nu} \implies |\lambda \cdot \nu_i| = |2b_i \cdot \nu_i - \sum_{j=1}^{n} x_{ij}\nu_j|$$
$$\geq |2b_i \cdot \nu_i| - |\sum_{j=1}^{n} x_{ij}\nu_i \geq 2b_i \cdot |\nu_i| - \sum_{j=1}^{n} x_{ij}|\nu_j||$$
$$\geq \underbrace{(2b_i - \sum_{j=1}^{n} x_{ij})}_{>0} |\nu_i| > 0 \implies |\lambda| > 0$$

Let $\overline{e}$ be a constant such that $\overline{e} > \max_i \frac{a_i}{2b_i - \sum_{j=1}^{n} x_{ij}}$. Consider game $G' = \left\{\mathcal{V}, \{u_i\}_{i\in\mathcal{V}}, \overline{A} = [0, \overline{e}]^n\right\}$. Note that $\overline{A}$ (the action space of game G') is convex and compact and utility $u_i(e_i, e_{-i})$ is concave in e_i. Therefore, by Brouwer fixed-point theorem, the best response mapping of game G' has at least one fixed point (Nash equilibrium). Let $\hat{\boldsymbol{e}}'$ be the

Nash equilibrium of game G', and $\hat{e}'_i$ be the maximum element in $\hat{\boldsymbol{e}}'$. By Equation (2), we know that $\hat{e}'_j \neq 0,\ \forall j$. We have,

$$\begin{aligned} \frac{d\ u_i(\boldsymbol{e})}{d\ e_i}|_{\boldsymbol{e}=\hat{\boldsymbol{e}}'} &\geq 0 \text{ (equality holds if } \hat{e}'_i < \overline{e}) \\ a_i - 2b_i\hat{e}'_i + \sum_{j=1}^n x_{ij}\hat{e}'_j &\geq 0 \implies \\ (2b_i - \sum_{j=1}^n x_{ij}) \cdot \hat{e}'_i &\leq a_i \implies \hat{e}'_i \leq \frac{a_i}{2b_i - \sum_{j=1}^n x_{ij}} < \overline{e}. \end{aligned}$$

Therefore, $\hat{e}'_i < \overline{e}$ which implies that $\hat{\boldsymbol{e}}'$ is an interior point of set $\overline{A}$ and it should be an NE for game G as well. Therefore, game G has at least one Nash equilibrium. By Equation (3), the fixed point of best response mapping of game $G(X)$ satisfies the following, $(2B - X) \cdot \hat{\boldsymbol{e}} = \boldsymbol{a}$. As $(2B - X)$ is invertible, the best response mapping has a unique fixed point $\hat{\boldsymbol{e}} = (2B - X)^{-1} \cdot \boldsymbol{a}$. As game $G(X)$ has at least one Nash equilibrium, and fixed point $(2B - X)^{-1} \cdot \boldsymbol{a}$ is the *only* candidate for NE, $(2B - X)^{-1} \cdot \boldsymbol{a}$ should be a non-negative vector and a unique NE for $G(X)$. ■

Proof. [Corollary 1] Let $\mathbf{0} \in \mathbb{R}^n$ be a zero vector. By Theorem 1, we know that $(2 \cdot B - X)^{-1} \cdot \tilde{\boldsymbol{a}} \succeq \mathbf{0}$ for any non-negative vector $\tilde{\boldsymbol{a}}$. Set $\tilde{a}_i = 1$ and $\tilde{a}_j = 0,\ \forall j \neq i$ and $\tilde{\boldsymbol{a}} = [\tilde{a}_1, \cdots, \tilde{a}_n]^T$. Then, $(2 \cdot B - X)^{-1} \cdot \tilde{\boldsymbol{a}} \succeq \mathbf{0}$ is the ith column of $(2 \cdot B - X)^{-1}$. Because i is arbitrary, all columns of $(2 \cdot B - X)^{-1}$ are non-negative. Moreover, we have,

$$\begin{aligned} (2B - X) \cdot \hat{\boldsymbol{e}} &= \boldsymbol{a} \\ (2B - X - \tilde{X}) \cdot \tilde{\boldsymbol{e}} &= \boldsymbol{a} \implies \\ \tilde{\boldsymbol{e}} = (2B - X)^{-1} \cdot \boldsymbol{a} &+ (2B - X)^{-1} \cdot \tilde{X} \cdot \tilde{\boldsymbol{e}} \\ &= \hat{\boldsymbol{e}} + \underbrace{(2B - X)^{-1} \cdot \tilde{X} \cdot \tilde{\boldsymbol{e}}}_{\succeq \mathbf{0}} \succeq \hat{\boldsymbol{e}} \end{aligned}$$

■

Proof. [Theorem 2] Define $f(\boldsymbol{e})$ as follows:

$$f(\boldsymbol{e}) = \sum_{i=1}^n u_i(\boldsymbol{e}) = \sum_{i=1}^n (-l_i + a_i \cdot e_i - b_i e_i^2 + e_i \cdot \sum_{j=1}^n x_{ij} e_j)$$

First, notice that the Hessian of $f(.)$ is $H = -2B + X + X^T$, and H is a symmetric matrix with real eigenvalues. Similar to the proof of Theorem 1, we can show that if $2b_i \geq \sum_{j=1}^n x_{ij} + x_{ji},\ \forall i$, then all eigenvalues of H are negative implying that $f(.)$ is strictly concave and H is invertible. Therefore, we can use the first order condition to find $\boldsymbol{e}^*$:

$$\begin{aligned} &\bigtriangledown f(\boldsymbol{e}^*) = \boldsymbol{a} - (2B - X - X^T) \cdot \boldsymbol{e}^* = 0 \implies \\ &\boldsymbol{e}^* = (2B - X - X^T)^{-1} \cdot \boldsymbol{a}\ . \end{aligned} \tag{17}$$

Note that $\boldsymbol{e}^* = (2B - X - X^T)^{-1} \cdot \boldsymbol{a}$ is the NE of game $G(X + X^T)$, which implies that $(2B - X - X^T)^{-1} \cdot \boldsymbol{a} \succeq \mathbf{0}$. The result then follows from Corollary 1. ■

Proof. [Theorem 8] Consider game G^1_{rp}. In this game $e_{1j} = e_{j1} = 0$ for all $j \in \mathcal{V} - \{1\}$. Let $\mathring{E} = [\mathring{e}_{ij}]_{n \times n}$ be the NE of G^1_{rp} with $\mathring{e}_{1j} = \mathring{e}_{j1} = 0,\ \forall j \in \mathcal{V} - \{1\}$. Moreover, let $\mathring{E}_i = \sum_{j=1}^n \mathring{e}_{ji}$. By the first order condition, best response of agent 1 is given by,

$$2b_1\mathring{e}_{11} - \sum_{j=1}^n x_{1j}\mathring{E}_j = a_1 \tag{18}$$

Moreover, by best response function of agent $i > 1$, we have,

$$2b_i\mathring{e}_{ii} - \sum_{j=1}^n x_{ij}\mathring{E}_j = a_i, \quad 2b_j\mathring{e}_{ij} - x_{ij}\mathring{E}_i = 0, j \neq i \tag{19}$$

Similar to Equation (6) and by Equations (18) and (19), $\left[\mathring{E}_1, \cdots, \mathring{E}_n\right]^T$ satisfies: $(2B - X^T_{[1]} - X) \cdot \left[\mathring{E}_1, \cdots, \mathring{E}_n\right]^T = \boldsymbol{a}$, where all elements of $X_{[1]}$ are equal to X except its first row and column which are zero vectors. Similar to Theorem 3, if $2b_i > \sum_{j=1}^n [x_{ij} + x_{ji}],\ \forall i$, then game G^1_{rp} has a unique Nash equilibrium, and we have,

$$\begin{aligned} \left[\mathring{E}_1, \cdots, \mathring{E}_n\right]^T &= (2B - X^T_{[1]} - X)^{-1}\boldsymbol{a} \\ \mathring{e}_{11} &= \frac{a_1}{2b_1} + \frac{\sum_{k=1}^n x_{1k} \cdot \mathring{E}_k}{2b_1} \\ \mathring{e}_{ii} &= \frac{a_i}{2b_i} + \frac{\sum_{k=1}^n x_{ik} \cdot \mathring{E}_k}{2b_i}\ \forall i > 1 \\ \mathring{e}_{ji} &= \frac{x_{ji} \cdot \mathring{E}_j}{2b_i}\ \forall j \neq i,\ i > 1,\ j > 1 \end{aligned} \tag{20}$$

By Corollary 1 and Equations (20) and (7), it is easy to see that $\mathring{E}_i \leq \hat{E}_i$ and $\mathring{e}_{ij} \leq \hat{e}_{ij},\ \forall i, j$.

$$\begin{aligned} v_1(\mathring{E}) &= -l_1 + a_1\mathring{e}_{11} - b_1(\mathring{e}_{11})^2 + (\mathring{e}_{11}) \sum_{j=1}^n x_{1j}\mathring{E}_j \\ &\leq -l_1 + a_1(\hat{e}_{11} + \sum_{j=2}^n \hat{e}_{j1}) - b_1\hat{e}^2_{11} \\ &+(\hat{e}_{11} + \sum_{j=2}^n \hat{e}_{j1}) \sum_{j=1}^n \left(x_{1j}(\sum_{k\neq 1} \hat{e}_{kj}) \right) \\ &\underbrace{\leq}_{\text{by definiton of NE for } G_{rp}} -l_1 + a_1(\hat{e}_{11} + \sum_{j=2}^n \hat{e}_{j1}) - \sum_{j=1}^n b_j\hat{e}^2_{1j} \\ &+(\hat{e}_{11} + \sum_{j=2}^n \hat{e}_{j1}) \sum_{j=1}^n x_{1j}\hat{E}_j = v_i(\hat{E}) \end{aligned} \tag{21}$$

Therefore, resource pooling satisfies voluntary participation wrt agent 1. We can show this holds wrt any agent. ■

Proof. [Theorem 9] The proof is similar to the proof of Theorem 8. Consider game $\overline{G}^1_{rp}$. We have,

$$\begin{aligned} \left[\acute{E}_1, \cdots, \acute{E}_n\right]^T &= (2B - X^T_{[r1]} - X)^{-1}\boldsymbol{a} \\ \acute{e}_{11} &= \frac{a_1}{2b_1} + \frac{\sum_{k=1}^n x_{1k} \cdot \acute{E}_k}{2b_1} \\ \acute{e}_{ii} &= \frac{a_i}{2b_i} + \frac{\sum_{k=1}^n x_{ik} \cdot \acute{E}_k}{2b_i}\ \forall i > 1 \\ \acute{e}_{ji} &= \frac{x_{ji} \cdot \acute{E}_j}{2b_i}\ \forall j \neq i,\ j > 1 \end{aligned} \tag{22}$$

where all the elements of $X_{[r1]}$ are equal to X except its first row which is a zero vector. By Corollary 1 and Equations (22) and (7), it is easy to see that $\acute{E}_i \leq \hat{E}_i$ and $\acute{e}_{ij} \leq \hat{e}_{ij},\ \forall i, j$. With the similar procedure as Equation (21), we can conclude that the resource pooling satisfies the strong voluntary participation defined in Definition 2 with respect to agent 1. Similarly, resource pooling satisfies the strong voluntary participation with respect to all agents. ■

Proof. [Theorem 10] Proof is similar to the proof of Theorem 3. We use the first order condition to find the best response functions.

$$\begin{aligned}\frac{\partial v_i(\boldsymbol{e}_i,\boldsymbol{e}_{-i})}{\partial e_{ii}} &= 0 \implies\\ \check{e}_{ii} &= \frac{a_i}{2b_i}+\frac{\sum_{k=1}^n x_{ik}\cdot \check{E}_k}{2b_i}\forall i\\ \frac{\partial v_j(\boldsymbol{e}_j,\boldsymbol{e}_{-j})}{\partial e_{ji}} &= 0 \implies\\ \check{e}_{ji} &= \frac{x_{ji}\cdot \check{E}_j}{2b_i}\ \forall j\neq i,\ j\in \bigcup_{k\in I(i)} C_k \quad (23)\end{aligned}$$

By adding above equations:

$$\begin{aligned}2b_i\cdot \check{E}_i &= a_i+\sum_{j=1}^n x_{ij}\check{E}_j+\sum_{j\in\bigcup_{k\in I(i)}C_k} x_{ji}\check{E}_j \quad \forall i\in\mathcal{V}\\ \implies \boldsymbol{a} &= (2B-X-X_c^T)\cdot\left[\check{E}_1,\cdots,\check{E}_n\right]^T \quad (24)\end{aligned}$$

Under assumption 2, $(2B-X-X_c^T)$ is invertible. We have,

$$(2B-X-X_c^T)^{-1}\cdot\boldsymbol{a} = \left[\check{E}_1,\cdots,\check{E}_n\right]^T \quad (25)$$

Moreover, by best response mapping we have,

$$\begin{aligned}\check{e}_{ii} &= \frac{a_i}{2b_i}+\frac{\sum_{j=1}^n x_{ij}\check{E}_j}{2b_i},\\ \check{e}_{ij} &= \frac{x_{ij}\cdot\check{E}_i}{2b_j},\forall j\neq i, j\in\bigcup_{k\in I(i)} C_k \quad (26)\end{aligned}$$

Therefore, the bast response mapping has a unique fixed point implying uniqueness of NE. ■

Proof. [Theorem 11]

$$\begin{aligned}\boldsymbol{e}^* &= (2B-X-X^T)^{-1}\cdot\boldsymbol{a}\\ \hat{\boldsymbol{e}} &= (2B-X)^{-1}\cdot\boldsymbol{a}\\ \left[\check{E}_1,\cdots,\check{E}_n\right]^T &= (2B-X-X_c^T)^{-1}\cdot\boldsymbol{a}\\ (2B-X) &\succeq (2B-X-X_c^T)\\ &\succeq (2B-X-X^T)\\ \text{Corollary 1} \implies \boldsymbol{e}^* &\succeq \left[\check{E}_1,\cdots,\check{E}_n\right]^T\succeq\hat{\boldsymbol{e}} \quad (27)\end{aligned}$$

Next we show that $v_i(\check{\boldsymbol{e}}_i,\check{\boldsymbol{e}}_{-i})\geq u_i(\hat{e}_i,\hat{e}_{-i})$. Let $\tilde{\boldsymbol{e}}_i$ be a vector with length $|\bigcup_{k\in\overline{I}(i)}C_k|$ and all its elements are zero except e_{ii} which is equal to $\hat{e}_i$ (effort level of agent i at NE of game G). By definition of NE, we have,

$$v_i(\check{\boldsymbol{e}}_i,\check{\boldsymbol{e}}_{-i})\geq v_i(\tilde{\boldsymbol{e}}_i,\check{\boldsymbol{e}}_{-i}). \quad (28)$$

As $\check{E}_i\geq\hat{e}_i,\ \forall i$, by Equations (26) and (3) we have $\check{e}_{ii}\geq\hat{e}_i$. Moreover,

$$\begin{aligned}&v_i(\tilde{\boldsymbol{e}}_i,\check{\boldsymbol{e}}_{-i}) = -l_i+a_i\cdot\hat{e}_i+a_i\sum_{k\neq i,k\in\bigcup_{r\in I(i)}C_r}\check{e}_{ki}-b_i\cdot(\hat{e}_i)^2\\ &+(\hat{e}_i+\sum_{k\neq i,k\in C_{I(i)}}\check{e}_{ki})\cdot\sum_{j=1}^n\left(x_{ij}\cdot(\sum_{k\neq i,k\in\bigcup_{r\in I(i)}C_r}\check{e}_{kj}\right)\geq\\ &-l_i+a_i\cdot\hat{e}_i-b_i\cdot(\hat{e}_i)^2+\hat{e}_i\cdot\sum_{j=1}^n x_{ij}\cdot\hat{e}_j = u_i(\hat{e}_i,\hat{e}_{-i}) \quad (29)\end{aligned}$$

By Equations (28) and (29), $v_i(\hat{E})\geq u_i(\hat{\boldsymbol{e}})\ \forall i\in V$. ■

Proof. [Theorem 12] Next we show $\check{\overline{E}}_i\geq\check{E}_i$. Note that $\left[\check{E}_1,\cdots,\check{E}_n\right]^T=(2B-X-X_c^T)^{-1}\cdot\boldsymbol{a}$ and $\left[\check{\overline{E}}_1,\cdots,\check{\overline{E}}_n\right]=(2B-X-\overline{X}_c^T)^{-1}\cdot\boldsymbol{a}$, where, entry (i,j) of $\overline{X}_c$ is equal to x_{ij} if agent i and j belong to the same community after merging community C_m and C_{m-1}. Otherwise, it is zero. We have,

$$\begin{aligned}(2B-X-X_c^T) &\succeq (2B-X-\overline{X}_c^T)\\ \text{Corollary 1}\implies\left[\check{\overline{E}}_1,\cdots,\check{\overline{E}}_n\right]^T &\succeq \left[\check{E}_1,\cdots,\check{E}_n\right] \quad (30)\end{aligned}$$

As $\check{\overline{E}}_i\geq\check{E}_i,\ \forall i$, by Equation (26), we have $\check{\overline{e}}_{ij}\geq\check{e}_{ij},\ \forall i,j$.

$$\begin{aligned}&v_i(\check{\boldsymbol{e}}_i,\check{\overline{\boldsymbol{e}}}_{-i}) = -l_i+a_i\cdot\check{e}_{ii}+a_i\sum_{k\neq i,k\in C_{\overline{I}(i)}}\check{\overline{e}}_{ki}-\sum_{j=1}^n b_j\cdot(\check{e}_{ij})^2\\ &+(\check{e}_{ii}+\sum_{k\neq i,k\in\bigcup_{r\in\overline{I}(i)}C_r}\check{\overline{e}}_{ki})\cdot\sum_{j=1}^n\left(x_{ij}\cdot(\check{e}_{ij}+\sum_{k\neq i,k\in\bigcup_{r\in\overline{I}(i)}C_r}\check{\overline{e}}_{kj})\right)\geq\\ &-l_i+a_i\cdot\check{E}_i-\sum_{j=1}^n b_j\cdot(\check{e}_{ij})^2+\check{E}_i\cdot\sum_{j=1}^n x_{ij}\cdot\check{E}_j = v_i(\check{\boldsymbol{e}}_i,\check{\boldsymbol{e}}_{-i})\end{aligned}$$

Moreover, by the definition of Nash equilibrium for game $\overline{G}_{rp}^c$, we have $v_i(\check{\overline{\boldsymbol{e}}}_i,\check{\overline{\boldsymbol{e}}}_{-i})\geq v_i(\check{\boldsymbol{e}}_i,\check{\overline{\boldsymbol{e}}}_{-i})$. Therefore, $v_i(\check{\overline{\boldsymbol{e}}}_i,\check{\overline{\boldsymbol{e}}}_{-i})\geq v_i(\check{\boldsymbol{e}}_i,\check{\boldsymbol{e}}_{-i})$. ■

B. On the mixed strategies Nash equilibrium in game G

We can show that there is no mixed strategies Nash equilibrium in game G. Assume that the action of agent i follows a probability distribution with mean value μ_i and variance σ_i^2. We have,

$$\mathbb{E}\{u_i(\boldsymbol{e})\}=\mathbb{E}\{a_ie_i-b_i\cdot e_i^2+e_i\sum_{j=1}^n x_{ij}e_j\} \quad (31)$$

$$=a_i\cdot\mu_i-b_i\cdot(\mu_i^2+\sigma_i^2)+\mu_i\cdot\mathbb{E}\{\sum_{j=1}^n x_{ij}e_i\} \quad (32)$$

As we can see in Equation (32), agent i always is able to improve its utility by setting $\sigma_i=0$. Therefore, a mixed strategy cannot be an optimal action at the NE. Therefore, the NE is always pure.

C. On the voluntary participation and budget balance constraints for taxation mechanisms

Neghizadeh and Liu [9] consider a model with a strictly concave utility function and show that the taxation mechanisms may not be able to satisfy the voluntary participation and budget balance constraints simultaneously. In this part, we provide an example to show that their result can be extended to the quadratic utility model. Consider an example with the following parameters,

$$n=30,\quad x_{ij}=1\ \forall i,j,\ i\neq j,\quad b_i=30,\ a_i=1,\ l_i=0\ \forall i.$$

In this example, the social welfare at the socially optimal outcome of game G is given by,

$$\begin{aligned}\boldsymbol{e}^* &= (2B-X-X^T)^{-1}\cdot\boldsymbol{a}=[0.5\dots0.5]^T,\\ u_i(\boldsymbol{e}^*) &= 0.25\ \forall i,\quad \sum_{i=1}^n u_i(\boldsymbol{e}^*)=7.5. \quad (33)\end{aligned}$$

By the notion of exit equilibrium defined in [9], the agents' effort when agent i unilaterally opts out of the taxation mechanism is given by,

$$\begin{aligned}\hat{e}_i^{(i)} &= \arg\max_{e_i \geq 0} u_i(e_i, \hat{e}_{-i}^{(i)}),\\ \hat{e}_{-i}^{(i)} &= \arg\max_{e_{-i} \succeq 0} \sum_{j \neq i} u_j(\hat{e}_i^{(i)}, e_{-i}),\end{aligned} \tag{34}$$

where, $\hat{e}_i^{(i)}$ is the effort of agent i, and $\hat{e}_{-i}^{(i)}$ is a $(n-1)$-dimensional vector denoting effort of the agents excluding agent i at the exit equilibrium. Using the first order condition, the solution to Equation (34) satisfies the following system of linear equations,

$$\begin{aligned}a_i - 2b_i\hat{e}_i^{(i)} + \sum_{k=1}^{n} x_{ik}\hat{e}_k^{(i)} &= 0,\\ a_j - 2b_j\hat{e}_j^{(i)} + \sum_{k=1}^{n} x_{jk}\hat{e}_k^{(i)} + \sum_{k \neq i} x_{kj}\hat{e}_k^{(i)} &= 0 \ \forall j \neq i,\end{aligned}$$

or equivalently, $(2B - X - X_{[i]}^T) \cdot \hat{\boldsymbol{e}}^{(i)} = \boldsymbol{a}$, where $\hat{\boldsymbol{e}}^{(i)} = [\hat{e}_1^{(i)}, \ldots, \hat{e}_n^{(i)}]^T$, and entry (r, s) of $X_{[i]}$ is equal to x_{rs} if $r \neq i$ and $s \neq i$. Otherwise, it is zero.

In our example, the utility of agent i when he is the outlier, and the other agents are participating in the taxation mechanism is given by,

$$\begin{aligned}\hat{\boldsymbol{e}}^{(i)} &= (2B - X - X_{[i]}^T)^{-1} \cdot \boldsymbol{a},\\ \hat{e}_i^{(i)} &= 0.1564, \hat{e}_j^{(i)} = 0.2891 \quad \forall j \neq i,\\ u_i(\hat{\boldsymbol{e}}^{(i)}) &= 0.7338, \ u_j(\hat{\boldsymbol{e}}^{(i)}) = 0.1672.\end{aligned}$$

By symmetry, it is easy to see that $u_i(\hat{\boldsymbol{e}}^{(i)}) = 0.7338, \forall i$. By [9], if $\sum_{i=1}^{n} u_i(\boldsymbol{e}^*) - \sum_{i=1}^{n} u_i(\hat{\boldsymbol{e}}^{(i)}) < 0$, then there is not any taxation mechanism which induces the socially optimal outcome and satisfies both week budget balance and voluntary participation constraints. In our example, we have $\sum_{i=1}^{n} u_i(\boldsymbol{e}^*) - \sum_{i=1}^{n} u_i(\hat{\boldsymbol{e}}^{(i)}) = -14.5143 < 0$ which shows the result in [9] can be extended to the quadratic utility model.

While taxation mechanisms cannot induce socially optimal outcome and address the free-riding issue in our model, resource pooling is able to do that.

References

[1] M. M. Khalili, X. Zhang, and M. Liu, "Incentivizing effort in interdependent security games using resource pooling," in *Proceedings of the 14th Workshop on the Economics of Networks, Systems and Computation*. ACM, 2019, p. 5.

[2] R. Zhang, Q. Zhu, and Y. Hayel, "A bi-level game approach to attack-aware cyber insurance of computer networks," *IEEE Journal on Selected Areas in Communications*, vol. 35, no. 3, pp. 779–794, 2017.

[3] H. Takabi, J. B. Joshi, and G.-J. Ahn, "Security and privacy challenges in cloud computing environments," *IEEE Security & Privacy*, vol. 8, no. 6, pp. 24–31, 2010.

[4] J. Chen and Q. Zhu, "Interdependent strategic security risk management with bounded rationality in the internet of things," *IEEE Transactions on Information Forensics and Security*, pp. 1–1, 2019.

[5] H. Varian, *System Reliability and Free Riding*. Boston, MA: Springer US, 2004, pp. 1–15. [Online]. Available: https://doi.org/10.1007/1-4020-8090-5_1

[6] J. Grossklags, S. Radosavac, A. A. Cárdenas, and J. Chuang, "Nudge: Intermediaries' role in interdependent network security," in *Trust and Trustworthy Computing*, A. Acquisti, S. W. Smith, and A.-R. Sadeghi, Eds. Berlin, Heidelberg: Springer Berlin Heidelberg, 2010, pp. 323–336.

[7] M. M. Khalili, P. Naghizadeh, and M. Liu, "Designing cyber insurance policies: The role of pre-screening and security interdependence," *IEEE Transactions on Information Forensics and Security*, vol. 13, no. 9, pp. 2226–2239, 2018.

[8] C. Ioannidis, D. Pym, and J. Williams, "Is public co-ordination of investment in information security desirable?" *Journal of Information Security*, vol. 7, no. 2, pp. 60–80, 2016.

[9] P. Naghizadeh and M. Liu, "Exit equilibrium: Towards understanding voluntary participation in security games," in *Computer Communications, IEEE INFOCOM 2016-The 35th Annual IEEE International Conference on*. IEEE, 2016, pp. 1–9.

[10] I. Vakilinia and S. Sengupta, "Fair and private rewarding in a coalitional game of cybersecurity information sharing," *IET Information Security*, 2019.

[11] W. Saad, T. Alpcan, T. Basar, and A. Hjorungnes, "Coalitional game theory for security risk management," in *2010 Fifth International Conference on Internet Monitoring and Protection*. IEEE, 2010, pp. 35–40.

[12] A. Galeotti, S. Goyal, M. O. Jackson, F. Vega-Redondo, and L. Yariv, "Network games," *The review of economic studies*, vol. 77, no. 1, pp. 218–244, 2010.

[13] N. Heydaribeni and A. Anastasopoulos, "Distributed mechanism design for multicast transmission," in *2018 IEEE Conference on Decision and Control (CDC)*. IEEE, 2018, pp. 4200–4205.

[14] R. A. Miura-Ko, B. Yolken, J. Mitchell, and N. Bambos, "Security decision-making among interdependent organizations," in *Computer Security Foundations Symposium, 2008. CSF'08. IEEE 21st*. IEEE, 2008, pp. 66–80.

[15] A. R. Hota and S. Sundaram, "Interdependent security games on networks under behavioral probability weighting," *IEEE Transactions on Control of Network Systems*, vol. 5, no. 1, pp. 262–273, March 2018.

[16] L. Jiang, V. Anantharam, and J. Walrand, "How bad are selfish investments in network security?" *IEEE/ACM Transactions on Networking (TON)*, vol. 19, no. 2, pp. 549–560, 2011.

[17] S. Amin, G. A. Schwartz, and S. S. Sastry, "Security of interdependent and identical networked control systems," *Automatica*, vol. 49, no. 1, pp. 186 – 192, 2013. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0005109812004682

[18] R. J. La, "Effects of degree correlations in interdependent security: Good or bad?" *IEEE/ACM Transactions on Networking*, vol. 25, no. 4, pp. 2484–2497, Aug 2017.

[19] O. Candogan, K. Bimpikis, and A. Ozdaglar, "Optimal pricing in networks with externalities," *Operations Research*, vol. 60, no. 4, pp. 883–905, 2012.

[20] F. Parise and A. Ozdaglar, "A variational inequality framework for network games: Existence, uniqueness, convergence and sensitivity analysis," *Games and Economic Behavior*, 2019. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0899825618301891

[21] J. Grossklags, N. Christin, and J. Chuang, "Secure or insure?: a game-theoretic analysis of information security games," in *Proceedings of the 17th international conference on World Wide Web*. ACM, 2008, pp. 209–218.

[22] J. de Marti and Y. Zenou, "Network games with incomplete information," *Journal of Mathematical Economics*, vol. 61, pp. 221 – 240, 2015. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0304406815001202

[23] R. Pal, L. Golubchik, K. Psounis, and P. Hui, "Security pricing as enabler of cyber-insurance a first look at differentiated pricing markets," *IEEE Transactions on Dependable and Secure Computing*, 2017.

[24] A. Matsui, "Best response dynamics and socially stable strategies," *Journal of Economic Theory*, vol. 57, no. 2, pp. 343–362, 1992.

[25] D. G. Luenberger, "Complete stability of noncooperative games," *Journal of Optimization Theory and Applications*, vol. 25, no. 4, pp. 485–505, 1978.

[26] G. H. Golub and C. F. Van Loan, *Matrix computations*. JHU press, 2012, vol. 3.

[27] *How Spotify makes use of partnerships*, http://www.digitalsocialstrategy.org/bac/2016/12/08/how-spotify-makes-use-of-partnerships/.

[28] *Amazon partners American Express on new business credit card launch*, https://ibsintelligence.com/ibs-journal/ibs-news/amazon-partners-american-express-on-new-business-credit-card-launch/.

[29] *Mastercard and Apple Taking on a Digital First Approach to Payments*, https://www.paymentsjournal.com/mastercard-apple-digital-approach-payments/.

[30] U. von Luxburg, "A tutorial on spectral clustering," *Statistics and Computing*, vol. 17, no. 4, pp. 395–416, Dec 2007. [Online]. Available: https://doi.org/10.1007/s11222-007-9033-z